

**CHEBx****PURPOSE**

Compute the Chebychev polynomial of the first kind and order x where x is an integer between 0 and 10.

**DESCRIPTION**

Chebychev polynomials are orthogonal polynomials defined by the following equation (the value of n specifies the order):

$$T_n(x) = \cos(n \arccos(x)) \quad (\text{EQ 6-82})$$

In practice, Chebychev polynomials are calculated as:

$$C_n(x) = \sum_{m=0}^n C_m x^m \quad (\text{EQ 6-83})$$

with the  $C_m$  coefficients being tabulated on page 795 of the Handbook of Mathematical Functions (see REFERENCE below).

DATAPLOT uses tabulated coefficients to compute the Chebychev polynomial for a given value of x.

Chebychev polynomials are only orthogonal in the interval (-1,1). Although DATAPLOT computes values for arguments to the Chebychev functions outside of this range, these are typically not of interest.

**SYNTAX 1**

LET <y2> = CHEB0(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 0.

**SYNTAX 2**

LET <y2> = CHEB1(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 1.

**SYNTAX 3**

LET <y2> = CHEB2(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 2.

**SYNTAX 4**

LET <y2> = CHEB3(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 3.

**SYNTAX 5**

LET <y2> = CHEB4(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 4.

## SYNTAX 6

LET <y2> = CHEB5(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 5.

## SYNTAX 7

LET <y2> = CHEB6(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 6.

## SYNTAX 8

LET <y2> = CHEB7(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 7.

## SYNTAX 9

LET <y2> = CHEB8(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 8.

## SYNTAX 10

LET <y2> = CHEB9(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 9.

## SYNTAX 11

LET <y2> = CHEB10(<y1>) <SUBSET/EXCEPT/FOR qualification>

where <y1> is a variable or a parameter;

<y2> is a variable or a parameter (depending on what <y1> is) where the computed Chebychev polynomial value is stored;  
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the Chebychev polynomial of order 10.

## EXAMPLES

LET A = CHEB3(-2)

LET X2 = CHEB2(X1)

LET X2 = CHE5(X1-4)

## NOTE

For order 0 (i.e.,  $n = 0$ ),  $T_0(x) = \cos(0) = 1$ .

For order 1 (i.e.,  $n = 1$ ),  $T_1(x) = \cos(\arccos(x)) = x$ .

For order 2 (i.e.,  $n = 2$ ),  $T_2(x) = \cos(2\arccos(x)) = 2x^2 - 1$ .

For order 3 (i.e.,  $n = 3$ ),  $T_3(x) = 4x^3 - 3x$ .

For order 4 (i.e.,  $n = 4$ ),  $T_4(x) = 8x^4 - 8x^2 + 1$ .

For order 5 (i.e.,  $n = 5$ ),  $T_5(x) = 16x^5 - 20x^3 + 5x$ .

For order 6 (i.e.,  $n = 7$ ),  $T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$ .

For order 7 (i.e.,  $n = 7$ ),  $T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$ .

For order 8 (i.e.,  $n = 8$ ),  $T_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$ .

For order 9 (i.e.,  $n = 9$ ),  $T_9(x) = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x$ .

For order 10 (i.e.,  $n = 10$ ),  $T_{10}(x) = 512x^{10} - 1280x^8 + 1120x^6 - 400x^4 + 50x^2 - 1$ .

#### DEFAULT

None

#### SYNONYMS

None

#### RELATED COMMANDS

BESS0	=	Compute the Bessel function of the first kind, order 0.
BESS1	=	Compute the Bessel function of the first kind, order 1.

#### REFERENCE

“Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55,” Abramowitz and Stegun, National Bureau of Standards, 1964 (Chapter 22).

#### APPLICATIONS

Special functions, approximating functions

#### IMPLEMENTATION DATE

Pre-1987

## PROGRAM

```

XLIMITS -1 1; XTIC OFFSET 0.1 0.1
YLIMITS -1 1; YTIC OFFSET 0.1 0.1
MULTIPLY 4 3; MULTIPLY CORNER COORDINATES 0 0 100 100
TITLE SIZE 3.5
TIC LABEL SIZE 3
LET STRING S = CHEB
LOOP FOR K = 0 1 10
  TITLE CHEBYCHEV POLYNOMIALS ORDER ^K
  LET STRING CHEB = ^S&^K
  PLOT ^CHEB(X) FOR X = -1 .05 1
END OF LOOP
END OF MULTIPLY

```

