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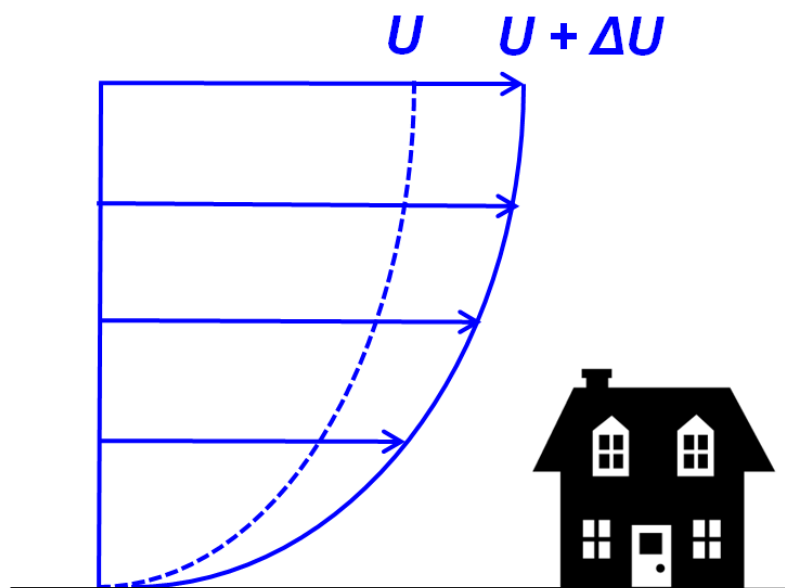**National Institute of Standards and Technology**

Technology Administration, U.S. Department of Commerce

Numerical Simulation of Along-Wind Loading on Small Structures Using a Simplified Wind Flow Model

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Abstract

Aerodynamic measurements of wind-induced pressures on structures are required for the codification of design wind loads. However, the reliable measurement of pressures on low-rise buildings in atmospheric boundary layer flow remains a challenge. Two major causes are the difficulty of simulating low-frequency turbulent fluctuations in boundary-layer wind tunnels and the small scale of models that can be accommodated in typical civil engineering aerodynamic testing facilities, especially for relatively small buildings such as residential homes. To address this issue, a simplified laboratory technique was recently proposed for the accurate and repeatable measurement of pressures on such buildings. The technique rests on the hypothesis that aerodynamic effects induced on small buildings by low-frequency fluctuations with high spatial coherence are equivalent to those induced by an increment in the mean wind speed. Preliminary measurements appear to have validated this hypothesis. The purpose of this study is to present an approximate numerical method for estimating the requisite increment in the mean wind speed. The method is based on the quasi-static representation of the pressures induced on the windward face of a rectangular building by wind normal to that face. The study provides a point of departure for the quantitative definition of simplified flows in future experimental studies.

Keywords: Building technology; simplified flow model; spatial coherence; atmospheric boundary layer; wind tunnel test

Acknowledgements

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1. Introduction

Aerodynamic measurements of wind-induced pressures on structures are required for the codification of design wind loads. However, the reliable measurement of pressures on low-rise buildings in atmospheric boundary layer (ABL) flow remains a challenge, as has been shown by the significant discrepancies among results obtained in different wind tunnel facilities (Fritz et al. 2008), or even in the same wind tunnel (Surry et al. 2003). Two major contributors of such discrepancies are the practical difficulty of simulating low-frequency turbulent fluctuations in the laboratory and the small scale of models that can be accommodated in typical civil engineering aerodynamic testing facilities, especially for relatively small buildings such as residential homes.

To address these issues, a simplified laboratory technique was proposed for the accurate and repeatable measurement of pressures on such buildings (Simiu et al. 2010). Instead of simulating an *ABL flow* including its low-frequency fluctuations, the simulation of a simplified wind flow model (i.e., *simplified flow*) was suggested. The simplified flow has a reference mean wind speed larger than the mean wind speed of the simulated conventional ABL flow, but does not contain the low-frequency fluctuations present in the ABL-like flow. This means that the energy of the missing low-frequency fluctuations is supplied, in the simplified flow, by the increment in mean wind speed, which may be regarded as a flow fluctuation with zero frequency. It is hypothesized that because low-frequency fluctuations are highly coherent spatially over small distances, the aerodynamic effects of the ABL-like flow are approximately equivalent to those induced by an augmented mean wind speed. The use of the proposed simplified flow can improve significantly the reliability and repeatability of pressure measurements in wind tunnel testing.

One question that needs to be addressed is: how large should the increment of the mean velocity be in order to provide a correct approximate substitute for the missing low-frequency fluctuations? This study proposes an approach to answering this question. We consider the simple case of the total wind force acting on the windward face of a rectangular building acted upon by wind normal to that face. For this case it is possible to calculate approximately that force both for flow nominally conforming to the conventional ABL model, and for flow conforming to the simplified model just described. The study also proposes an answer to the following question: what is the definition of "low-frequency fluctuations"? The answers based on the present study are intended to provide guidance required for aerodynamic testing of small buildings in simplified flows.

Since the contributions to the turbulence intensity and the integral turbulence scale are overwhelmingly due to low-frequency fluctuations, ABL flows in wind tunnel simulations impose a constraint on the model scale. For simplified flows this constraint does not exist, and the model scale depends only on wind tunnel blockage considerations.

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2. Methodology

This study is part of an effort aimed at establishing the feasibility of measuring pressures on small buildings by using a simplified flow, described in Sect. 1, in lieu of a conventional ABL-like flow commonly developed in wind tunnel laboratories. This study uses a quasi-static analytical model for the description of along-wind pressures on the windward face of rectangular buildings subjected to flow with mean speed normal to that face, with a view to achieving two goals: (1) estimate an appropriate upper limit frequency f_{low} of low-frequency fluctuations eliminated in the simplified flow, and (2) estimate the corresponding values of the increased mean wind speed $c\bar{U} = \bar{U} + \Delta\bar{U}$ where $c > 1$. To this end this study first examines the spatial coherence of the wind pressures due to low-frequency fluctuations on the windward wall of small buildings, and estimates the frequency f_{low} below which that coherence is sufficiently large for those pressures to be replaceable, without significant errors, by the pressures due to the increment in mean wind speed $\Delta\bar{U}$.

This study considers a low-rise building with rectangular shape (width b, height h), in terrain with open exposure immersed in the ABL wind flow (Fig. 1). The wind speed $U(y, z, t)$ is assumed to vary with width y, height z, and time t, and consists of the mean wind speed $\bar{U}(z)$ and the wind speed longitudinal fluctuations about the mean, $u(y, z, t)$. The velocity $U(y, z, t)$ is assumed to be normal to the wider face of the building.
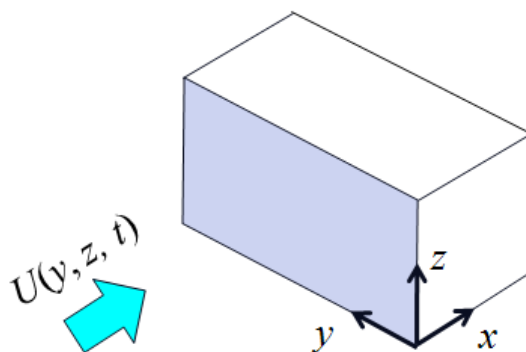

Figure 1. Schematic of the building (height h, width b)

To calculate the peak total aerodynamic force F_{peak} induced on the windward face of a building by (1) the ABL flow and (2) the simplified flow, the following steps are used:

Step 1: Estimation of peak force F_{peak} induced by the ABL flow on the windward building face.

A calculation of the peak total aerodynamic force F_{peak} is performed under the following assumptions:

1. The spectral density of the longitudinal flow fluctuations u is described by the expression for the modified Kaimal spectrum:

$$\frac{n\,S_u(z,n)}{u_*^2} = \frac{200f}{(1+50f)^{5/3}} \tag{1}$$

where f is the reduced frequency corresponding to a frequency n, defined as $nz/\bar{U}(z)$ and u_* is the friction velocity (Simiu and Scanlan, 1996, p. 59). This expression is valid for frequencies $0 < f \le f_{cut-off}$ in which it is reasonable to assume a cut-off frequency $f_{cut-off} = 10$ (i.e., $S_u(z, n)$ = 0 for $f > f_{cut-off}$).

2. The expression for the spatial coherence of the longitudinal wind velocity fluctuations u is

$$\text{Coh}(y_1,y_2,z_1,z_2,n) = \exp\left\{-\frac{n\left[C_y^2(y_1-y_2)^2 + C_z^2(z_1-z_2)^2\right]^{1/2}}{\frac{1}{2}\left[\bar{U}(z_1)+\bar{U}(z_2)\right]}\right\} \tag{2}$$

where n denotes the frequency, C_y = 16 and C_z = 10 are reasonable estimates of exponential decay coefficients in the y and z directions, and (y_1, z_1) and (y_2, z_2) are the coordinates of two points on the windward wall, respectively (e.g., Simiu and Scanlan, 1996, p. 65).

3. The longitudinal flow fluctuations and the flow-induced forces on the windward wall are approximately Gaussian.

Using these assumptions, the total wind-induced peak force F_{peak} on the windward wall can be expressed as the sum of the mean force and the peak force due to all fluctuations:

$$F_{peak} \approx \bar{F}_{\bar{U}} + \kappa_{F'}\sigma_{F'} \tag{3}$$

where

$$\bar{F}_{\bar{U}} = \int_0^h \int_0^b \frac{1}{2}\rho C_p \bar{U}^2(z)\,dy\,dz \tag{4}$$

b is the width of the building, h is the height of the building, ρ is the air density, $C_p = \bar{p}(z)/\left[\frac{1}{2}\rho\bar{U}^2(z)\right] \approx 0.8$ is the mean pressure coefficient where $\bar{p}(z)$ is the mean pressure at height z, $\kappa_{F'}$ is the peak factor, and $\sigma_{F'}$ is the r.m.s. of the fluctuating force F'.

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The peak factor for a flow with a duration of T seconds is approximately (Davenport 1964)

$$\kappa_{F'} \approx \sqrt{2\ln(v_{F'}T)} + \frac{0.577}{\sqrt{2\ln(v_{F'}T)}}$$

$$\text{where} \quad v_{F'} = \left[\frac{\int_0^{n_{cut-off}} n^2 S_{F'}dn}{\int_0^{n_{cut-off}} S_{F'}dn}\right]^{1/2} \tag{5}$$

where $v_{F'}$ is the average cycling rate for the peak force, $S_{F'}$ is the spectral density of the fluctuating force F' on the windward wall, and $n_{cut-off}$ is the dimensional cut-off frequency corresponding to $f_{cut-off}$ (Eq. 1). The r.m.s. of the fluctuating force F' is obtained by integration as follows:

$$\sigma_{F'} =$$
$$\left[\int_0^{n_{cut-off}} \int_0^h \int_0^h \int_0^b \int_0^b \rho^2 C_p^{\ 2}\overline{U(z_1)}\,\overline{U(z_2)}\,S_u^{\ 1/2}(z_1,n)\,S_u^{\ 1/2}(z_2,n)\times \atop \text{Coh}(y_1,y_2,z_1,z_2,n)\,dy_1\,dy_2\,dz_1\,dz_2\,dn \right]^{\frac{1}{2}} \tag{6}$$

In the above equation $\text{Coh}(y_1,y_2,z_1,z_2,n)$ is the spatial coherence function of the fluctuating wind speeds at two locations as defined in Eq. (2). This completes the calculation of the peak force F_{peak} induced by the ABL flow.

Step 2: Estimation of peak force F_{peak1} induced by the simplified flow.

The estimation process is similar to Step 1 except that:

1. The spectral density of the longitudinal velocity fluctuations u in the simplified flow is

$$S_u(z,n) = 0 \qquad \text{for } 0 < f \le f_{low}$$

$$\frac{n S_u(z,n)}{u_*^2} = \frac{200f}{(1+50f)^{5/3}} \quad \text{for } f_{low} < f \le f_{cut-off} \tag{7}$$

where f_{low} can be selected near the lower limit of the interval within which the Kolmogorov inertial subrange hypothesis holds in the ABL wind, and $f_{cut-off} = 10$ as explained earlier. Recall that the reduced frequency f is based on mean wind speed $\overline{U}(z)$.

The simplified flow has no low-frequency fluctuating part (A in Fig. 2) as expressed in Eq. (7), and has an increased mean speed $c\overline{U}$, which is required so that the peak force generated by the ABL flow (with speed $\overline{U}$ and spectral content denoted by A and B in the figure) be the same as the peak force generated by the simplified flow (with speed $c\overline{U}$ and spectral content denoted by B in the figure). Note that wind-induced pressures on buildings are also affected by high-frequency fluctuations, which should be simulated in the simplified flow.

The peak force due to the simplified flow is approximately equal to the sum of mean force due to the increased mean speed and the peak force induced by the high-frequency fluctuations:

$$F_{peak1} \approx \overline{F}_{c\overline{U}} + \kappa_{F'high}\sigma_{F'high} \tag{8}$$

where the mean force $\overline{F}_{c\overline{U}}$ is based on the increased mean wind speed $c\overline{U}$ and has an expression similar to Eq. (4), and the peak factor $\kappa_{F'high}$ and the r.m.s. of the fluctuating force $\sigma_{F'high}$ are calculated for the high frequency fluctuations $f_{low} < f \le f_{cut-off}$, and have expressions similar to Eqs. (5 and 6).
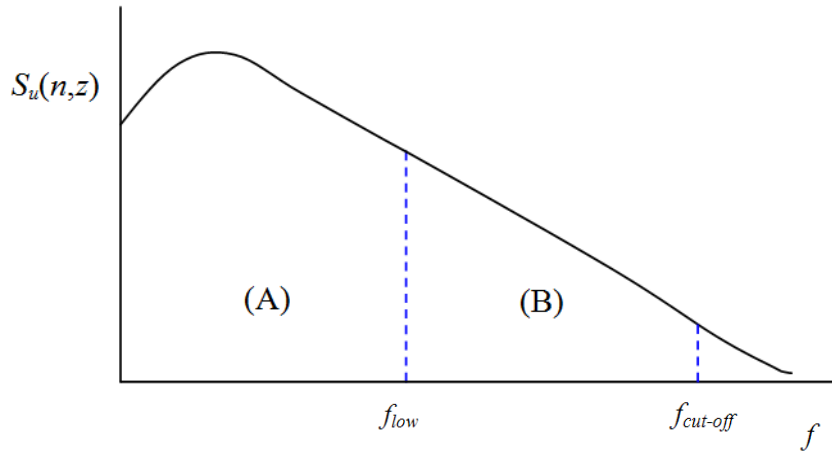

Figure 2. Spectrum of longitudinal fluctuating wind (not to scale)

Step 3: Estimation of the upper limit of low-frequency fluctuations f_{low}.

To generate approximately equivalent peak forces due to the ABL flow (Step 1) and the simplified flow (Step 2), the low-frequency fluctuations must have sufficiently high spatial coherence that they can be replaced by the additional mean force due to the increment in speed $\Delta\overline{U}$.

Step 4, variant (a): Estimation of increased mean wind speed $c\overline{U}$.

Given f_{low}, the mean wind speed $c\overline{U} = \overline{U} + \Delta\overline{U}$ can be determined by equating the peak force due to the ABL flow and the peak force due to the simplified flow (i.e., $F_{peak} = F_{peak1}$). The requisite factor c and the corresponding mean wind speed increment $\Delta\overline{U}$ are therefore estimated as follows:

$$\overline{F}_{c\overline{U}} = c^2\overline{F}_{\overline{U}} \tag{9}$$

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$$c = \sqrt{\frac{\kappa_{F'}\sigma_{F'} - \kappa_{F'high}\sigma_{F'high}}{\bar{F}_{\bar{U}}} + 1} \qquad (10)$$

$$\Delta\bar{U} = (c-1)\bar{U} \qquad (11)$$

The numerical implementation of the calculation is shown in the Appendix (Sect. A. 3).

Step 4, variant (b): Simplified estimation of increased mean wind speed $c'\bar{U}$.

An alternative estimate of the increased speed, denoted by $c'\bar{U}$, can be performed by equating the peak wind speed due to the low-frequency fluctuations in the ABL flow and the increment in the mean speed $\Delta\bar{U}'$ in the simplified flow. The results are then

$$\bar{U} + \kappa_u\sigma_u = c'\bar{U} + \kappa_{uhigh}\sigma_{uhigh} \qquad (12)$$

$$c' = \frac{\kappa_u\sigma_u - \kappa_{uhigh}\sigma_{uhigh}}{\bar{U}} + 1 \qquad (13)$$

$$\Delta\bar{U}' = \kappa_u\sigma_u - \kappa_{uhigh}\sigma_{uhigh} \qquad (14)$$

where κ_u and σ_u are the peak factor and the r.m.s. of the longitudinally fluctuating wind speed corresponding to all frequency fluctuations $0 < f \leq f_{cut-off}$, and κ_{uhigh} and σ_{uhigh} are their counterparts corresponding to high frequency $f_{low} < f \leq f_{cut-off}$.

The calculated $\Delta\bar{U}'$ is slightly more conservative (i.e., larger) and less accurate than $\Delta\bar{U}$ calculated in Step 4(a). The larger the building, the less accurate the simplified calculation is. The numerical implementation of the simplified calculation is shown in the Appendix (Sect. A. 4).

3. Application

We considered rectangular buildings with height $h = 12$ m and various widths ($b = 6$ m to 22 m). First, we investigated the extent to which the imperfect coherence of the low-frequency pressures induced by the ABL flow is significant in practice. This investigation was performed by calculating the ratio

$$R_{low} = \frac{\kappa_{F'-low}\sigma_{F'-low}\big|_{C_y=16,\,C_z=10}}{\kappa_{F'-low}\sigma_{F'-low}\big|_{C_y=0,\,C_z=0}} \tag{15}$$

where the numerator and the denominator are, respectively, the peak force due to the low-frequency fluctuating flow fluctuations based on the use of exponential decay coefficients of $C_y = 16$ and $C_z = 10$, and $C_y = 0$ and $C_z = 0$. The various terms in Eq. (15) are calculated by using Eqs. (2, 5, and 6) for the frequency range $0 < f \leq f_{low}$. The details of the calculation are provided in the Appendix (Sect. A. 5). We emphasize that the exponential decay coefficients exhibit significant variability in nature, so the values selected for this study are not definitive.

Figure 3 is a plot of R_{low} as a function of b and f_{low}. As expected, for lower f_{low} and lower b the value of R_{low} is closer to unity. A lower f_{low} results in better pressure simulations in the simplified wind. However, a higher f_{low} is more desirable from an experimental viewpoint, since low-frequency fluctuations are difficult to simulate. In this study, $f_{low} = 0.1$, which is close to the approximate practical lower limit of the frequency range within which Kolmogorov's hypothesis concerning the inertial subrange holds (Simiu and Scanlan 1996, p. 57).
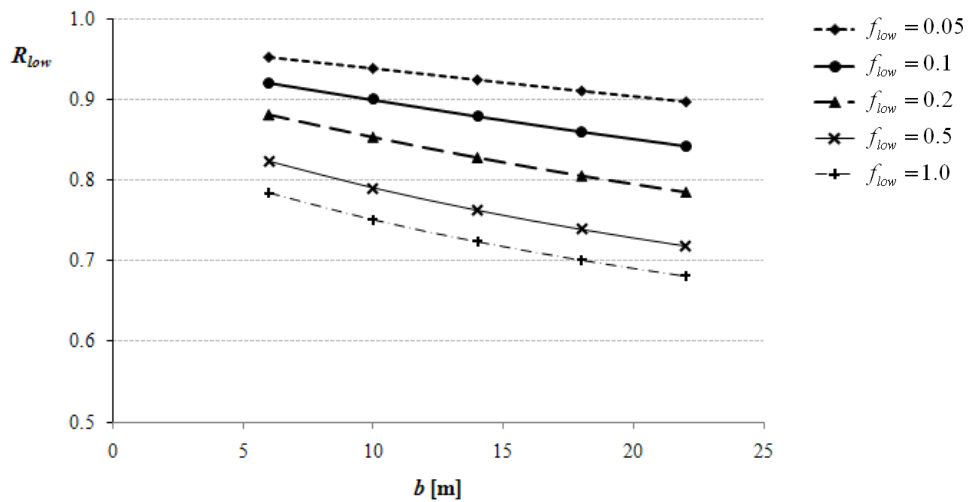

Figure 3. Spatial coherence of peak force due to low-frequency fluctuations

Two estimation methods of the increased mean speed were used (see Steps 4(a) and 4(b)). Figure 4 shows the ratio of the increment in the mean speed to the reference wind speed at eave height (Eqs. 11 and 14). The ratios do not change significantly as the width increases. For example, the ratio based on Step 4(a) is 0.29, which means that the simplified wind of $f_{low} = 0.1$ requires a 29 % increase in the mean speed to predict the same peak forces on the windward wall of the building as in the ABL wind. The ratio based on Step 4(b) is 0.33, which is less accurate and more conservative than the ratio from Step 4(a) because it is based on the equality between the peak wind speeds in both wind models, rather than on the equality between the respective peak forces. The difference is approximately 10 %, and the corresponding differences between the respective estimated peak forces are less than 5 %. Both methods, therefore, are acceptable; however, the estimation method of Step 4(b) is more practical and straightforward.

All of the results reported so far correspond to open terrain exposure (roughness length $z_0 = 0.03$ m). The increased mean wind speeds in the simplified wind flow were also estimated for open water exposure ($z_0 = 0.005$ m), suburban exposure ($z_0 = 0.3$ m), and urban terrain exposure ($z_0 = 0.7$ m). Figure 5 shows that for open water surface exposure the ratios $\Delta\bar{U}/\bar{U}$ are 0.22 (corresponding to estimates based on equivalent peak forces, Step 4(a)) and 0.25 (corresponding to estimates based on equivalent peak speeds, Step 4(b)). For the urban terrain, they are 0.64 and 0.69, respectively. It is evident that both methods can be used, regardless of terrain exposure.

The analytical approach presented in this section can be used to provide approximate estimates of the augmented mean wind speeds $c\bar{U}$ to be employed in experimental work. Refinements of these estimates would depend on experimental results similar to those of Simiu et al. (2010).
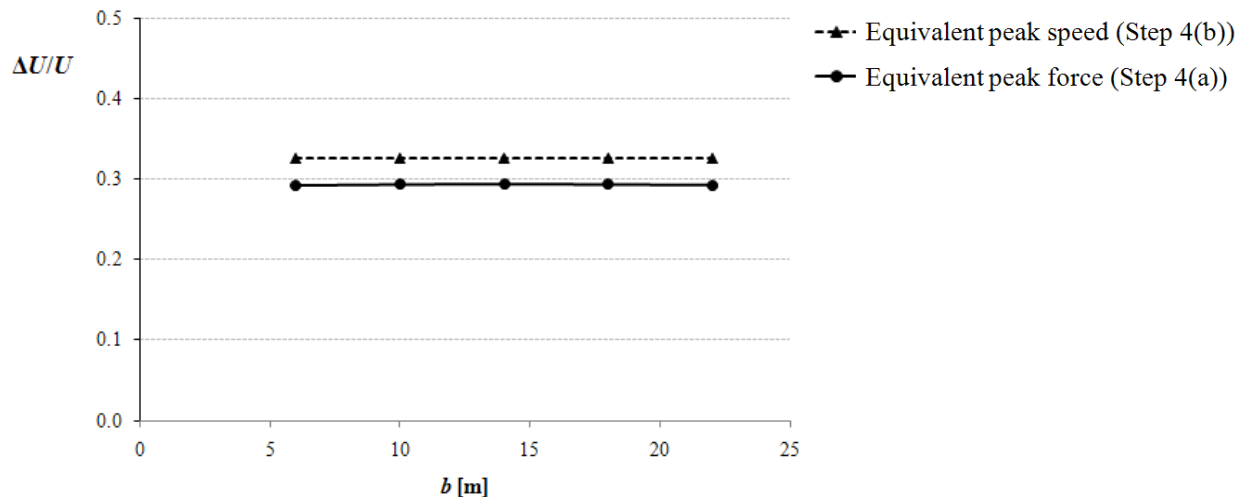

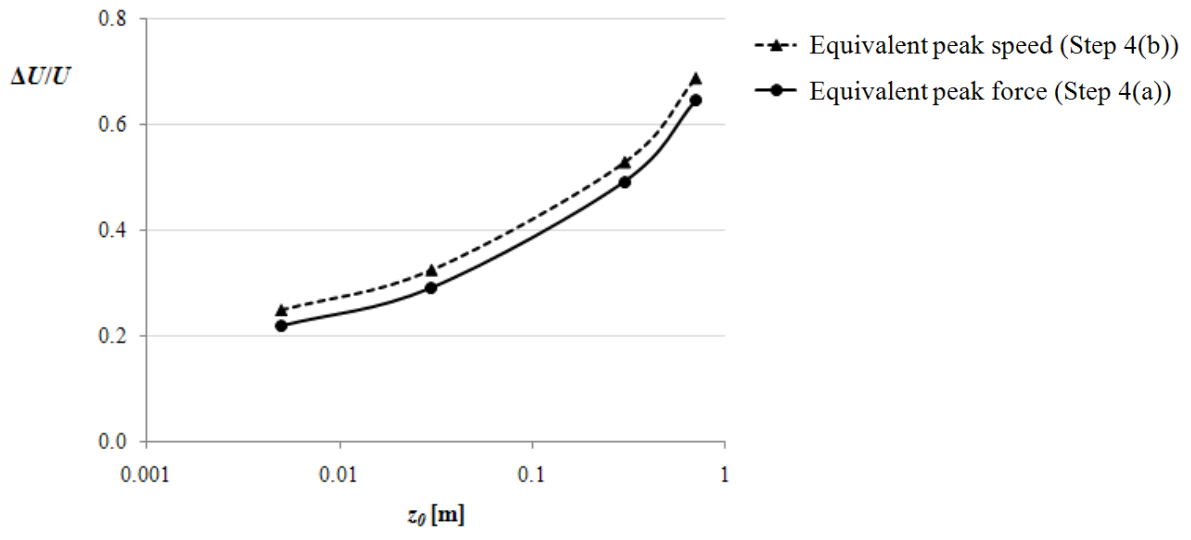Figure 4. Ratio $\Delta\bar{U}/\bar{U}$ ($f_{low} = 0.1$)

Figure 5. Ratio $\Delta\bar{U}/\bar{U}$ as a function of terrain conditions

4. Conclusions

This study provided an approach to estimating analytically the two parameters required to define simplified wind flows: the increment in mean wind speed $\Delta\bar{U}$ and the upper limit of low-frequency fluctuations f_{low}. The results of the study suggest that the hypothesis according to which a simplified flow can be substituted for an ABL-like flow is acceptable for sufficiently small buildings, at least as far as the simulation of wind forces on the windward face of small buildings normal to the direction of the mean wind speed is concerned. The low-frequency fluctuations whose upper limit frequency is less than f_{low} = 0.1 have sufficiently high spatial coherence to be replaceable, with acceptably small errors, by an augmented mean wind speed. Based on the estimation using equivalent peak force, the increments in mean wind speed of the simplified wind are from approximately 20 % to 70 % of the original reference mean wind speed, depending upon terrain exposure. The estimation methods suggested in this study are applicable to any theoretical or empirical spectrum. Note that testing in simplified flows has the advantage of allowing geometric scales to be larger than is possible in ABL-like flows. The methodology and results presented in this report provide a point of departure for a quantitative definition of simplified flows in future experimental studies.

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Appendix

Three MATHEMATICA programs have been developed

estimation_wind_equiv_F.nc,
estimation_wind_equiv_U.nc, and
estimation_spatial_coherence.nc.

The first and the second program estimate increments in mean wind speed of a simplified wind using the equivalent peak force and the equivalent peak wind speed, respectively. The third program estimates the spatial coherence of pressures on a windward wall induced by the low-frequency fluctuation.

A.1 Inputs

The inputs to all programs consist of the:

1) Building information
   • Building height (zh)
   • Building width (y1min, y1max: the far left and the far right coordinate of the building in y, see Fig. 1)
2) Wind information
   • Wind speed at 10 m above the ground (U10)
   • Roughness length (z0)
   • Pressure coefficient (Cp)
   • Air density (rho)
   • Exponential decay coefficients (Cy, Cz)
   • Cut-off frequency (nmax)
   • Upper limit of low frequency (nlow)

A.2 outputs

The output of 'estimation_wind_equiv_F.nc' consists of the:
   • Ratio of increment ΔU to original U (RatioU)

The output of 'estimation_wind_equiv_U.nc' consists of the:
   • Ratio of increment ΔU to original U (RatioU)
   • Total peak force from the simplified flow (FTpeakhigh)

The output of 'estimation_spatial_coherence.nc' consists of the:
   • Standard deviation of low-frequency fluctuating force (Cy = 16, Cz = 10)
   • Mean force
   • Peak force from low-frequency fluctuations
   • Total peak force from low-frequency fluctuations
   • Ratio of standard deviation (Cy = 16, Cz = 10) to standard deviation (Cy = Cz = 0)

- Ratio of peak low-frequency fluctuating force ($Cy = 16$, $Cz = 10$) to peak low-frequency fluctuating force ($Cy = Cz = 0$)
- Ratio of total peak force ($Cy = 16$, $Cz = 10$) to total peak force ($Cy = Cz = 0$).

A.3 MATHEMATICA program: 'estimation_wind_equiv_F.nc'

```
(* Eave height [m] *)
zh = 12;
(* Wind speed at the 10 m above ground *)
U10=55;
(* Roughness length [m] *)
z0=0.03;
(* von Karman coefficient *)
k =0.4;
(* Friction velocity *)
ustar=k*U10/Log[10/z0];
(* Wind speed at the eave height *)
Uzh=1/k*ustar*Log[zh/z0];
(* Pressure coefficient *)
Cp=0.8;
(* air density [kg/m^3] *)
rho = 1.25;
(* Exponential decaying coefficients *)
Cy = 16;
Cz = 10;
(* Range in y *)
y1min=0;
y1max= 14;
y2min =y1min;
y2max =y1max;
z1min= z0;
z1max= zh;
z2min =z1min;
z2max =z1max;

(* n of cut-off frequency *)
nmax=10*Uzh/zh ;     (* cut-off f = 10 *)
(* n of low frequency *)
nlow=0.1*Uzh/zh ;     (* low f = 0.1 *)

(* Integration *)
P = 200*rho^2*Cp^2*ustar^3/k*
NIntegrate[
Sqrt[z1*z2*Log[z1/z0]*Log[z2/z0]/(1+50*n*k*z1/ustar/Log[z1/z0])^(5/3)/(1+50*n*k*z2/ustar/Log[z2/z0])^(5/3)]*
Exp[-2*k*n*Sqrt[Cz^2*(z1-z2)^2+Cy^2*(y1-y2)^2]/ustar/(Log[z1/z0]+Log[z2/z0])],
{y1,y1min,y1max},{y2,y2min,y2max},{z1,z1min,z1max},{z2,z2min,z2max},{n,0,nmax},MaxRecursion->20,Method-
>{GlobalAdaptive,MaxErrorIncreases->10000}];


Pn = 200*rho^2*Cp^2*ustar^3/k*
NIntegrate[n^2*
Sqrt[z1*z2*Log[z1/z0]*Log[z2/z0]/(1+50*n*k*z1/ustar/Log[z1/z0])^(5/3)/(1+50*n*k*z2/ustar/Log[z2/z0])^(5/3)]*
Exp[-2*k*n*Sqrt[Cz^2*(z1-z2)^2+Cy^2*(y1-y2)^2]/ustar/(Log[z1/z0]+Log[z2/z0])],
{y1,y1min,y1max},{y2,y2min,y2max},{z1,z1min,z1max},{z2,z2min,z2max},{n,0,nmax},MaxRecursion->20,Method-
>{GlobalAdaptive,MaxErrorIncreases->10000}];


Phigh = 200*rho^2*Cp^2*ustar^3/k*
NIntegrate[
Sqrt[z1*z2*Log[z1/z0]*Log[z2/z0]/(1+50*n*k*z1/ustar/Log[z1/z0])^(5/3)/(1+50*n*k*z2/ustar/Log[z2/z0])^(5/3)]*
Exp[-2*k*n*Sqrt[Cz^2*(z1-z2)^2+Cy^2*(y1-y2)^2]/ustar/(Log[z1/z0]+Log[z2/z0])],
```

```
{y1,y1min,y1max},{y2,y2min,y2max},{z1,z1min,z1max},{z2,z2min,z2max},{n,nlow,nmax},MaxRecursion->20,Method-
>{GlobalAdaptive,MaxErrorIncreases->10000}];

Phighn = 200*rho^2*Cp^2*ustar^3/k*
NIntegrate[n^2*
Sqrt[z1*z2*Log[z1/z0]*Log[z2/z0]/(1+50*n*k*z1/ustar/Log[z1/z0])^(5/3)/(1+50*n*k*z2/ustar/Log[z2/z0])^(5/3)]*
Exp[-2*k*n*Sqrt[Cz^2*(z1-z2)^2+Cy^2*(y1-y2)^2]/ustar/(Log[z1/z0]+Log[z2/z0])],
{y1,y1min,y1max},{y2,y2min,y2max},{z1,z1min,z1max},{z2,z2min,z2max},{n,nlow, nmax},MaxRecursion->20,Method-
>{GlobalAdaptive,MaxErrorIncreases->10000}];


(* Standard deviation of fluctuating force *)
(* over all freq.*)
sigmaF     =Sqrt[P];
sigmaFn     =Sqrt[Pn];
(* for the high freq.*)
sigmaFhigh = Sqrt[Phigh];
sigmaFhighn     =Sqrt[Phighn];

(* Peak factor *)
NuF = sigmaFn/sigmaF;
KF = Sqrt[2*Log[NuF*3600]]+0.577/Sqrt[2*Log[NuF*3600]];
NuFhigh = sigmaFhighn/sigmaFhigh;
KFhigh = Sqrt[2*Log[NuFhigh*3600]]+0.577/Sqrt[2*Log[NuFhigh*3600]];

(* Mean force *)
AF = 0.5*rho*Cp/k^2*NIntegrate[(Log[z/z0])^2,{y,y1min,y1max},{z,z1min,z1max}];
Fmean = ustar^2*AF;

(* Peak force *)
Fpeak = KF*sigmaF;
Fpeakhigh = KFhigh*sigmaFhigh;

(* Total peak force *)
FTpeak = Fmean+Fpeak;
FTpeakhigh = Fmean+Fpeakhigh;

(* Increased meain wind speed *)
Print["Ratio of increment delU to original U"]
RatioU = Sqrt[(Fpeak-Fpeakhigh)/Fmean+1]-1
```

A.4 MATHEMATICA program: 'estimation_wind_equiv_U.nc'

```
(* Eave height [m] *)
zh = 12;
(* Wind speed at the 10 m above ground *)
U10=55;
(* Roughness length [m] *)
z0=0.03;
(* von Karman coefficient *)
k =0.4;
(* Friction velocity *)
ustar=k*U10/Log[10/z0];
(* Wind speed at the eave height *)
Uzh=1/k*ustar*Log[zh/z0];
(* Pressure coefficient *)
Cp=0.8;
(* air density [kg/m^3] *)
rho = 1.25;
(* Exponential decaying coefficients *)
Cy = 16;
```

```
Cz = 10;
(* Range in y *)
y1min=0;
y1max= 14;
y2min =y1min;
y2max =y1max;
z1min= z0;
z1max= zh;
z2min =z1min;
z2max =z1max;

(* n of cut-off frequency *)
nmax=10*Uzh/zh ;     (* cut-off f = 10 *)
(* n of low frequency *)
nlow=0.1*Uzh/zh ;     (* low f = 0.1 *)

(* Integration *)
U = 200*k*ustar*
NIntegrate[zh/Log[zh/z0]/(1+50*n*k*zh/ustar/Log[zh/z0])^(5/3),
{n,0,nmax},MaxRecursion->20,Method->{GlobalAdaptive,MaxErrorIncreases->10000}];

Un = 200*k*ustar*
NIntegrate[n^2*zh/Log[zh/z0]/(1+50*n*k*zh/ustar/Log[zh/z0])^(5/3),
{n,0,nmax},MaxRecursion->20,Method->{GlobalAdaptive,MaxErrorIncreases->10000}];

Uhigh = 200*k*ustar*
NIntegrate[zh/Log[zh/z0]/(1+50*n*k*zh/ustar/Log[zh/z0])^(5/3),
{n,nlow,nmax},MaxRecursion->20,Method->{GlobalAdaptive,MaxErrorIncreases->10000}];

Uhighn = 200*k*ustar*
NIntegrate[n^2*zh/Log[zh/z0]/(1+50*n*k*zh/ustar/Log[zh/z0])^(5/3),
{n,nlow,nmax},MaxRecursion->20,Method->{GlobalAdaptive,MaxErrorIncreases->10000}];

(* Standard deviation of fluctuating force *)
sigmaU     =Sqrt[U];
sigmaUhigh = Sqrt[Uhigh];
sigmaUn     =Sqrt[Un];
sigmaUhighn     =Sqrt[Uhighn];

(* Peak factor *)
NuU = sigmaUn/sigmaU;
KU = Sqrt[2*Log[NuU*3600]]+0.577/Sqrt[2*Log[NuU*3600]];
NuUhigh = sigmaUhighn/sigmaUhigh;
KUhigh = Sqrt[2*Log[NuUhigh*3600]]+0.577/Sqrt[2*Log[NuUhigh*3600]];


(* Peak speede *)
Upeak = KU*sigmaU;
Upeakhigh = KUhigh*sigmaUhigh;

(* increment of U *)
delU = Upeak-Upeakhigh;

Print["Ratio of increment delU to original U"]
RatioU = delU/Uzh
c = RatioU +1;

(* Estimation of peak force in the simplified wind*)
Phigh = 200*rho^2*Cp^2*ustar^3/k*
NIntegrate[
Sqrt[z1*z2*Log[z1/z0]*Log[z2/z0]/(1+50*n*k*z1/ustar/Log[z1/z0])^(5/3)/(1+50*n*k*z2/ustar/Log[z2/z0])^(5/3)]*
Exp[-2*k*n*Sqrt[Cz^2*(z1-z2)^2+Cy^2*(y1-y2)^2]/ustar/(Log[z1/z0]+Log[z2/z0])],
```

```
{y1,y1min,y1max},{y2,y2min,y2max},{z1,z1min,z1max},{z2,z2min,z2max},{n,nlow,nmax},MaxRecursion->20,Method-
>{GlobalAdaptive,MaxErrorIncreases->10000}];

Phighn = 200*rho^2*Cp^2*ustar^3/k*
NIntegrate[n^2*
Sqrt[z1*z2*Log[z1/z0]*Log[z2/z0]/(1+50*n*k*z1/ustar/Log[z1/z0])^(5/3)/(1+50*n*k*z2/ustar/Log[z2/z0])^(5/3)]*
Exp[-2*k*n*Sqrt[Cz^2*(z1-z2)^2+Cy^2*(y1-y2)^2]/ustar/(Log[z1/z0]+Log[z2/z0])],
{y1,y1min,y1max},{y2,y2min,y2max},{z1,z1min,z1max},{z2,z2min,z2max},{n,nlow, nmax},MaxRecursion->20,Method-
>{GlobalAdaptive,MaxErrorIncreases->10000}];

(* Mean force *)
AF = 0.5*rho*Cp/k^2*NIntegrate[(Log[z/z0])^2,{y,y1min,y1max},{z,z1min,z1max}];
Fmean = c^2*ustar^2*AF;

(* Standard deviation of fluctuating force *)
sigmaFhigh = Sqrt[Phigh];
sigmaFhighn     =Sqrt[Phighn];

(* Peak factor *)
NuFhigh = sigmaFhighn/sigmaFhigh;
KFhigh = Sqrt[2*Log[NuFhigh*3600]]+0.577/Sqrt[2*Log[NuFhigh*3600]];

(* Peak force *)
Fpeakhigh = KFhigh*sigmaFhigh;

(* Total peak force *)
Print["Total peak force from the simplified wind"]
FTpeakhigh = Fmean+Fpeakhigh
```

A.5 MATHEMATICA program: 'estimation_spatial_coherence.nc'

```
y2max =y1max;
z1min= z0;
z1max= zh;
z2min =z1min;
z2max =z1max;

(* n of cut-off frequency *)
nmax=10*Uzh/zh ;     (* cut-off f = 10 *)
(* n of low frequency *)
nlow=0.1*Uzh/zh ;     (* low f = 0.1 *)

(* Integration for Cy=16, Cz=10 *)
P = 200*rho^2*Cp^2*ustar^3/k*
NIntegrate[
Sqrt[z1*z2*Log[z1/z0]*Log[z2/z0]/(1+50*n*k*z1/ustar/Log[z1/z0])^(5/3)/(1+50*n*k*z2/ustar/Log[z2/z0])^(5/3)]*
Exp[-2*k*n*Sqrt[Cz^2*(z1-z2)^2+Cy^2*(y1-y2)^2]/ustar/(Log[z1/z0]+Log[z2/z0])],
{y1,y1min,y1max},{y2,y2min,y2max},{z1,z1min,z1max},{z2,z2min,z2max},{n,0,nlow},MaxRecursion->20,Method-
>{GlobalAdaptive,MaxErrorIncreases->10000}];

Pn = 200*rho^2*Cp^2*ustar^3/k*
NIntegrate[n^2*
Sqrt[z1*z2*Log[z1/z0]*Log[z2/z0]/(1+50*n*k*z1/ustar/Log[z1/z0])^(5/3)/(1+50*n*k*z2/ustar/Log[z2/z0])^(5/3)]*
Exp[-2*k*n*Sqrt[Cz^2*(z1-z2)^2+Cy^2*(y1-y2)^2]/ustar/(Log[z1/z0]+Log[z2/z0])],
{y1,y1min,y1max},{y2,y2min,y2max},{z1,z1min,z1max},{z2,z2min,z2max},{n,0,nlow},MaxRecursion->20,Method-
>{GlobalAdaptive,MaxErrorIncreases->10000}];


(* Integration for Cy=Cz=0 *)
P0 = 200*rho^2*Cp^2*ustar^3/k*
```

```
NIntegrate[
Sqrt[z1*z2*Log[z1/z0]*Log[z2/z0]/(1+50*n*k*z1/ustar/Log[z1/z0])^(5/3)/(1+50*n*k*z2/ustar/Log[z2/z0])^(5/3)],
{y1,y1min,y1max},{y2,y2min,y2max},{z1,z1min,z1max},{z2,z2min,z2max},{n,0,nlow},MaxRecursion->20,Method-
>{GlobalAdaptive,MaxErrorIncreases->10000}];

Pn0 = 200*rho^2*Cp^2*ustar^3/k*
NIntegrate[n^2*
Sqrt[z1*z2*Log[z1/z0]*Log[z2/z0]/(1+50*n*k*z1/ustar/Log[z1/z0])^(5/3)/(1+50*n*k*z2/ustar/Log[z2/z0])^(5/3)],
{y1,y1min,y1max},{y2,y2min,y2max},{z1,z1min,z1max},{z2,z2min,z2max},{n,0,nlow},MaxRecursion->20,Method-
>{GlobalAdaptive,MaxErrorIncreases->10000}];


(* Standard deviation of fluctuating force *)
Print["Standard deviation of low-freq. fluctuating force (Cy=16, Cz=10)"]
sigmaF     =Sqrt[P]
sigmaFn    =Sqrt[Pn];
Print["Standard deviation of low-freq.fluctuating force (Cy=Cz=0)"]
sigmaF0    =Sqrt[P0]
sigmaFn0    =Sqrt[Pn0];

(* Peak factor *)
Print["Calculating peak factor (Cy=16, Cz=10)"]
NuF = sigmaFn/sigmaF;
KF = Sqrt[2*Log[NuF*3600]]+0.577/Sqrt[2*Log[NuF*3600]]
Print["Calculating peak factor (Cy=Cz=0)"]
NuF0 = sigmaFn0/sigmaF0;
KF0 = Sqrt[2*Log[NuF0*3600]]+0.577/Sqrt[2*Log[NuF0*3600]]


(* Mean force *)
AF = 0.5*rho*Cp/k^2*NIntegrate[(Log[z/z0])^2,{y,y1min,y1max},{z,z1min,z1max}];
Print["Mean force"]
Fmean = ustar^2*AF
Print[" "]

(* Peak force *)
Print["Peak force from low-freq. fluctuations (Cy=16, Cz=10)"]
Fpeak     = KF*sigmaF
Print["Peak force from low-freq. fluctuations (Cy=Cz=0)"]
Fpeak0 = KF0*sigmaF0
Print[" "]

(* Total peak force *)
Print["Total peak force from low-freq. fluctuations (Cy=16, Cz=10)"]
FTpeak     = Fmean+Fpeak
Print["Total peak force from low-freq. fluctuations (Cy=Cz=0)"]
FTpeak0 = Fmean+Fpeak0
Print[" "]

(* Ratio for checking the coherence *)
Print["Ratio of standard deviation (Cy = 16, Cz = 10) to standard deviation (Cy = Cz = 0)"]
Rst = sigmaF/sigmaF0
Print["Ratio of peak low-frequency fluctuating force (Cy = 16, Cz = 10) to peak low-frequency fluctuating force (Cy =
Cz = 0)"]
Rf     = Fpeak/Fpeak0
Print["Ratio of total peak force (Cy = 16, Cz = 10) to total peak force (Cy = Cz = 0)"]
Rtf     = FTpeak/FTpeak0
```