## CBESSI

## PURPOSE

Compute either the real or complex component of the modified Bessel function of the first kind and order v for a complex argument where v is a non-negative real number.

## DESCRIPTION

The modified Bessel function of the first kind with order v ( v is a non-negative real number) can be defined as:

$$
\begin{equation*}
I_{v}(z)=\left(\frac{z}{2}\right)^{v} \sum_{k=0}^{\infty} \frac{\left(\frac{z^{2}}{4}\right)^{k}}{k!\Gamma(v+k+1)} \tag{EQAux-52}
\end{equation*}
$$

where z is a complex number, $\Gamma$ is the Gamma function and ! is the factorial function.
The real part of the input argument must be less than the logarithm of the largest single precision number on the given computer. In addition, the input number times its complex conjugate cannot be zero. The order is restricted to values between 0 and 100 .

## SYNTAX 1

LET <y2> = CBESSIR(<r1>,<i1>,<v>)
<SUBSET/EXCEPT/FOR qualification>
where $\langle r 1>$ is the real component of a number, variable or parameter;
<i1> is the complex component of a number, variable or parameter;
$\langle\mathrm{v}\rangle$ is a non-negative number, variable, or parameter that specifies the order of the Bessel function;
< $\mathrm{y} 2>$ is a variable or a parameter (depending on what <r1> and <i1> are) where the computed Bessel value is stored;
and where the <SUBSET/EXCEPT/FOR qualification> is optional.
This syntax computes the real component.

## SYNTAX 2

LET <y2> = CBESSII(<r1>,<i1>,<v>)
<SUBSET/EXCEPT/FOR qualification>
where $\langle r 1\rangle$ is the real component of a number, variable or parameter;
<il> is the complex component of a number, variable or parameter;
$\langle\mathrm{v}\rangle$ is a non-negative number, variable, or parameter that specifies the order of the Bessel function;
< $\mathrm{y} 2>$ is a variable or a parameter (depending on what <r1> and <i1> are) where the computed Bessel value is stored;
and where the <SUBSET/EXCEPT/FOR qualification> is optional.
This syntax computes the complex component.

## EXAMPLES

LET X2 $=\operatorname{CBESSIR}(2,1,2)$
LET A = CBESSII $(2,2.5 .5)$
$\operatorname{LET} A=\operatorname{CBESSIR}(\mathrm{R} 1, \mathrm{C} 1,3)$
LET A $=\operatorname{CBESSII}(\mathrm{R} 1, \mathrm{C} 1,3)$

## NOTE 1

DATAPLOT uses the routine BESICF from the BESPAK library. This library was written by David Sagin (Sookne), Computer Center, Tel Aviv University.

## NOTE 2

Although DATAPLOT does not allow negative orders, negative orders can be calculated with the following relation:

$$
\begin{equation*}
\mathrm{I}_{-v}(x)=\mathrm{I}_{v}(x)+\frac{2}{\pi} \sin (v \pi) \mathrm{K}_{v}(x) \tag{EQAux-53}
\end{equation*}
$$

where z is a complex number and $\mathrm{K}_{\mathrm{v}}$ is the modified Bessel function of the third kind. The functions CBESSIR, CBESSII, CBESSKR, and CBESSKI can be used to compute the relevant Bessel functions.

## DEFAULT

None

## SYNONYMS

None

## RELATED COMMANDS

| BESSIN | $=$ | Compute the modified Bessel function of order N and real argument. |
| :---: | :---: | :---: |
| CBESSJR | $=$ | Compute the real component of the Bessel function of the first kind, order N , and complex argument. |
| CBESSJI | = | Compute the complex component of the Bessel function of the first kind, order N, and complex argument. |
| CBESSYR | $=$ | Compute the real component of the Bessel function of the second kind, order N, and complex argument. |
| CBESSYI | = | Compute the complex component of the Bessel function of the second kind, order N , and complex argument. |
| CBESSKR | $=$ | Compute the real component of the modified Bessel function of the third kind, order N , and complex argument. |
| CBESSKI | $=$ | Compute the complex component of the modified Bessel function of the third kind, order N , and complex argument. |

## REFERENCE

"Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55," Abramowitz and Stegun, National Bureau of Standards, 1964 (pages 355-433).
"Note on Backward Recurrence Algorithms," Olver and Sookne, Mathematics of Computation, Volume 26, October 1972.
"Recurrence Techniques for the Calculation of Bessel Functions," Goldstein and Thaler, Mathematics of Computation, Volume 13, April 1959.
"Bessel Functions of Complex Argument and Integer Order," Sookne, Journal of Research of the National Bureau of Standards, Series B, Volume 77A, July-December, 1973.

## APPLICATIONS

Special Functions
IMPLEMENTATION DATE 94/9

## PROGRAM

LET XR = SEQUENCE . 1.15
LET XC = SEQUENCE .1.15
LET ORDER = 2
LET YR = CBESSIR(XR,XC,ORDER)
LET YI = CBESSII(XR,XC,ORDER)
PRINT XR XC YR YI

